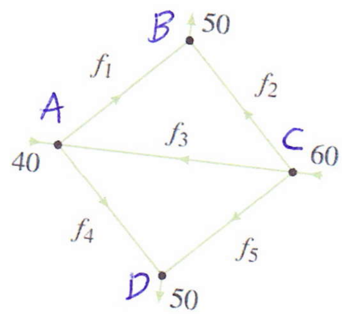
Math 260: Linear Algebra
Sections 1.4-1.6 Homework Problems

Book Problems: Section 1.4: #1a

Exercise 1.4.1 Find the possible flows in the following networks of pipes.

a.



A: $40 + f_3 = f_1 + f_4$

B: $f_1 + f_2 = 50$

C: $60 = f_5 + f_3 + f_2$

D: $f_4 + f_5 = 50$

$$\Rightarrow \begin{array}{l} f_1 - f_3 + f_4 = 40 \\ f_1 + f_2 = 50 \\ f_2 + f_3 + f_5 = 60 \\ f_4 + f_5 = 50 \end{array}$$

$$\Rightarrow \begin{array}{c} \begin{matrix} f_1 & f_2 & f_3 & f_4 & f_5 & \end{matrix} \\ \left[\begin{array}{ccccc|c} 1 & 0 & -1 & 1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 50 \\ 0 & 1 & 1 & 0 & 1 & 60 \\ 0 & 0 & 0 & 1 & 1 & 50 \end{array}\right] \end{array} \xrightarrow{RREF} \begin{array}{c} \begin{matrix} f_1 & f_2 & f_3 & f_4 & f_5 & \end{matrix} \\ \left[\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & -1 & -10 \\ 0 & 1 & 1 & 0 & 1 & 60 \\ 0 & 0 & 0 & 1 & 1 & 50 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$f_3 = r$

$f_5 = t$

$f_1 - f_3 - f_5 = -10 \Rightarrow f_1 = f_3 + f_5 - 10 = r + t - 10$

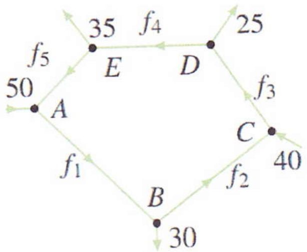
$f_2 + f_3 + f_5 = 60 \Rightarrow f_2 = 60 - f_3 - f_5 = 60 - r - t$

$f_4 + f_5 = 50 \Rightarrow f_4 = 50 - f_5 = 50 - t$

$$\boxed{\begin{array}{ll} f_1 = r + t - 10 & \\ f_2 = 60 - r - t & \\ f_3 = r & r, t \in \mathbb{R} \\ f_4 = 50 - t & \\ f_5 = t & \end{array}}$$

Book Problems: Section 1.4: #3

Exercise 1.4.3 A traffic circle has five one-way streets, and vehicles enter and leave as shown in the accompanying diagram.



a. Compute the possible flows.

b. Which road has the heaviest flow?

a)

A: $50 + f_5 = f_1$
B: $f_1 = f_2 + 30$
C: $f_2 + 40 = f_3$
D: $f_3 = f_4 + 25$
E: $f_4 = f_5 + 35$

$$\Rightarrow \begin{aligned} f_1 \qquad\qquad\qquad\qquad - f_5 &= 50 \\ f_1 - f_2 \qquad\qquad\qquad\qquad &= 30 \\ -f_2 + f_3 \qquad\qquad\quad &= 40 \\ f_3 - f_4 \qquad\quad &= 25 \\ f_4 - f_5 &= 35 \end{aligned}$$

$$\Rightarrow \begin{array}{c} \begin{array}{ccccc} f_1 & f_2 & f_3 & f_4 & f_5 \end{array} \\ \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & 50 \\ 1 & -1 & 0 & 0 & 0 & 30 \\ 0 & -1 & 1 & 0 & 0 & 40 \\ 0 & 0 & 1 & -1 & 0 & 25 \\ 0 & 0 & 0 & 1 & -1 & 35 \end{array}\right] \end{array}$$

RREF $\Rightarrow$

$$\begin{array}{c} \begin{array}{ccccc} f_1 & f_2 & f_3 & f_4 & f_5 \end{array} \\ \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & 50 \\ 0 & 1 & 0 & 0 & -1 & 20 \\ 0 & 0 & 1 & 0 & -1 & 60 \\ 0 & 0 & 0 & 1 & -1 & 35 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

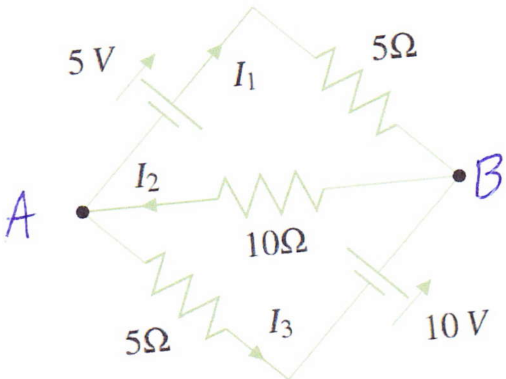
$f_5 = t$
$f_1 - f_5 = 50 \Rightarrow f_1 = f_5 + 50 = t + 50$
$f_2 - f_5 = 20 \Rightarrow f_2 = f_5 + 20 = t + 20$
$f_3 - f_5 = 60 \Rightarrow f_3 = f_5 + 60 = t + 60$
$f_4 - f_5 = 35 \Rightarrow f_4 = f_5 + 35 = t + 35$

So
$f_1 = t + 50$
$f_2 = t + 20$
$f_3 = t + 60$ $\quad t \geq 0$
$f_4 = t + 35$
$f_5 = t$

b) Since $f_3 = t + 60$ is the largest answer of all of the flows, the road connecting point C to point D has the largest flow.

Book Problems: Section 1.5: #2

Find the currents in the circuit



Junction A: $I_2 = I_1 + I_3$

Junction B: $I_1 + I_3 = I_2$

Top Loop: $5 - 5I_1 - 10I_2 = 0$

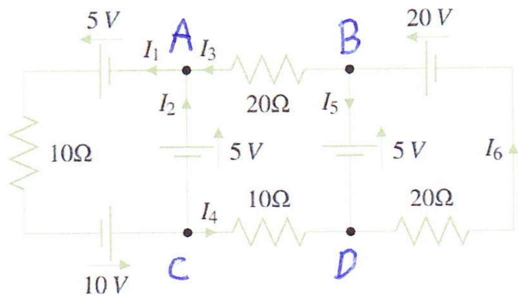
Bottom Loop: $10 - 10I_2 - 5I_3 = 0$

$$\Rightarrow \begin{aligned} I_1 - I_2 + I_3 &= 0 \\ I_1 - I_2 + I_3 &= 0 \\ 5I_1 + 10I_2 \quad &= 5 \\ 10I_2 + 5I_3 &= 10 \end{aligned} \Rightarrow \begin{array}{ccc} I_1 & I_2 & I_3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 5 & 10 & 0 & 5 \\ 0 & 10 & 5 & 10 \end{array}\right] \xrightarrow{RREF} \begin{array}{ccc} I_1 & I_2 & I_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -0.2 \\ 0 & 1 & 0 & 0.6 \\ 0 & 0 & 1 & 0.8 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \boxed{\begin{aligned} I_1 &= -0.2 \text{ Amps} \\ I_2 &= 0.6 \text{ Amps} \\ I_3 &= 0.8 \text{ Amps} \end{aligned}}$$

Book Problems: Section 1.5: #3

Find the currents in the circuit



Junction A $\Rightarrow$ $I_2 + I_3 = I_1$, or $I_1 - I_2 - I_3 = 0$.

Junction B $\Rightarrow$ $I_6 = I_3 + I_5$, or $I_3 + I_5 - I_6 = 0$

Junction C $\Rightarrow$ $I_1 = I_2 + I_4$, or $I_1 - I_2 - I_4 = 0$

Junction D $\Rightarrow$ $I_4 + I_5 = I_6$, or $I_4 + I_5 - I_6 = 0$

Left small Loop $\Rightarrow$ $10 + 5 + 5 - 10 I_1 = 0$, or $10 I_1 = 20$

Middle small loop $\Rightarrow$ $-10 I_4 + 5 - 20 I_3 - 5 = 0$, or $20 I_3 + 10 I_4 = 0$

Right small loop $\Rightarrow$ $-20 I_6 + 20 - 5 = 0$, or $20 I_6 = 15$

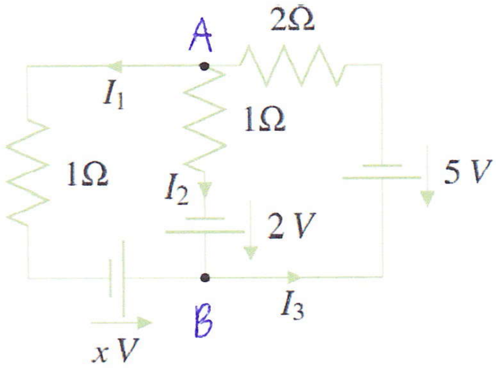
$$\begin{array}{cccccc} I_1 & I_2 & I_3 & I_4 & I_5 & I_6 \end{array}$$

$$\left[\begin{array}{cccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 10 & 0 & 0 & 0 & 0 & 0 & 20 \\ 0 & 0 & 20 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 20 & 15 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0.75 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0.75 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$I_1 = 2$ Amps
$I_2 = 2$ Amps
$I_3 = 0$ Amps
$I_4 = 0$ Amps
$I_5 = 0.75$ Amps
$I_6 = 0.75$ Amps

Book Problems: Section 1.5: #5

Find the voltage x such that the current $I_1 = 0$.



$I_1 = 0$

Junction A: $I_3 = I_1 + I_2$

Junction B: $I_1 + I_2 = I_3$

Left loop: $-1I_1 + x - 2 + 1I_2 = 0$

Right loop: $-5 - 2I_3 - 1I_2 + 2 = 0$

$$\Rightarrow \begin{aligned} I_1 \qquad\qquad &= 0 \\ I_1 + I_2 - I_3 \qquad &= 0 \\ I_1 + I_2 - I_3 \qquad &= 0 \\ -I_1 + I_2 \qquad + x &= 2 \\ -I_2 - 2I_3 \qquad &= 3 \end{aligned}
\Rightarrow
\begin{array}{c}
\begin{array}{ccccc} I_1 & I_2 & I_3 & x & \end{array} \\
\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 2 \\ 0 & -1 & -2 & 0 & 3 \end{array}\right]
\end{array}
\Rightarrow
\begin{array}{c}
\begin{array}{ccccc} I_1 & I_2 & I_3 & x & \end{array} \\
\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]
\end{array}$$

This system does have a solution and the 4th row says that

$\boxed{X = 3 \text{ Volts}}$

Book Problems: Section 1.6: #1

Balance the chemical reaction $CH_4 + O_2 \to CO_2 + H_2O$

$w\,CH_4 + x\,O_2 \to y\,CO_2 + z\,H_2O$

C: $w = y$

H: $4w = 2z$

O: $2x = 2y + z$

$$\Rightarrow \begin{array}{l} w \quad\quad -y \quad\quad = 0 \\ 4w \quad\quad\quad -2z = 0 \\ \quad 2x - 2y - z = 0 \end{array} \Rightarrow \begin{array}{cccc|c} w & x & y & z & \\ \end{array} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$$

RREF

$$\Rightarrow \begin{array}{cccc} w & x & y & z \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1/2 & 0 \end{array}\right]$$

$z = t$

$w - \frac{1}{2}z = 0 \Rightarrow w = \frac{1}{2}z = \frac{1}{2}t$

$x - z = 0 \Rightarrow x = z = t$

$y - \frac{1}{2}z = 0 \Rightarrow y = \frac{1}{2}z = \frac{1}{2}t$

$\Rightarrow \begin{array}{l} w = \frac{1}{2}t \\ x = t \\ y = \frac{1}{2}t \\ z = t \end{array}$

But, t must be a positive whole number, and each of w, x, y, z must all be positive whole numbers. Pick the smallest t that accomplishes this, which is when $t = 2$. So $w = 1$, $x = 2$, $y = 1$, $z = 2$

$$\Rightarrow \boxed{1\,CH_4 + 2\,O_2 \to 1\,CO_2 + 2\,H_2O}$$

Book Problems: Section 1.6: #2

Balance the chemical reaction $NH_3 + CuO \to N_2 + Cu + H_2O$

$v\,NH_3 + w\,CuO \to x\,N_2 + y\,Cu + z\,H_2O$

N: $v = 2x$

H: $3v = 2z$

Cu: $w = y$

O: $w = z$

$$\Rightarrow \begin{matrix} v & & -2x & & & = 0 \\ 3v & & & & -2z & = 0 \\ & w & & -y & & = 0 \\ & w & & & -z & = 0 \end{matrix} \Rightarrow \begin{array}{c} \begin{matrix} v & w & x & y & z & \end{matrix} \\ \left[\begin{array}{ccccc|c} 1 & 0 & -2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \end{array}\right] \end{array}$$

RREF

$$\Rightarrow \begin{array}{c} \begin{matrix} v & w & x & y & z & \end{matrix} \\ \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right] \end{array}$$

$z = t$

$v - \frac{2}{3}z = 0 \Rightarrow v = \frac{2}{3}z = \frac{2}{3}t$

$w - z = 0 \Rightarrow w = z = t$

$x - \frac{1}{3}z = 0 \Rightarrow x = \frac{1}{3}z = \frac{1}{3}t$

$y - z = 0 \Rightarrow y = z = t$

so

$v = \frac{2}{3}t$

$w = t$

$x = \frac{1}{3}t$

$y = t$

$z = t$

But t must be a positive whole number, and v, w, x, y, z must also be positive whole numbers. The minimum value of t that accomplishes this is $t = 3 \Rightarrow v = 2,\ w = 3,\ x = 1,\ y = 3,\ z = 3$.

So

$$\boxed{2NH_3 + 3CuO \to 1N_2 + 3Cu + 3H_2O}$$